



# 科技與數學教育 回顧與展望

施淑娟

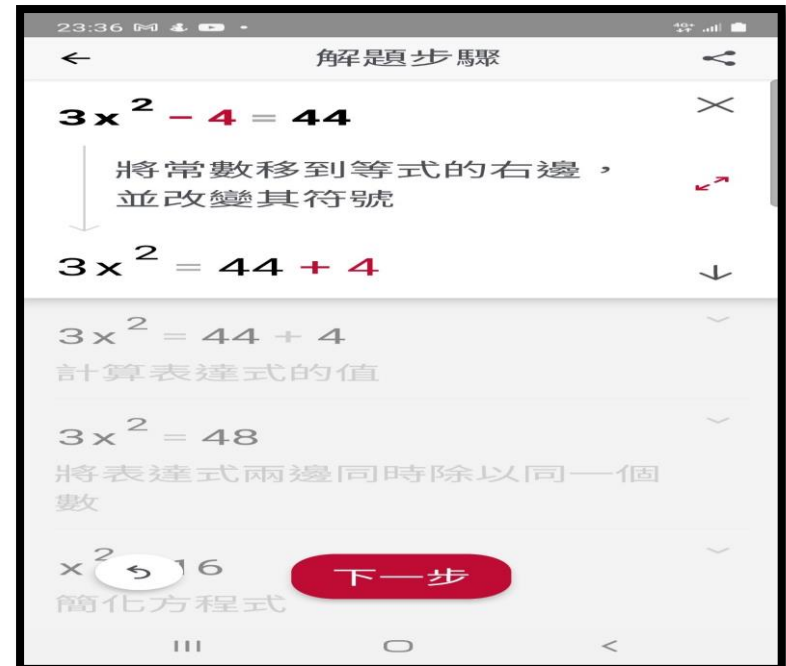
國立臺中教育大學教育資訊與測驗統計研究所



110年科技部數學教育學門成果討論會  
110.11.26



# 幾個數學學習科技的例子



**使用二次方程式求解。**

$$3x^2 - 44 = 4$$

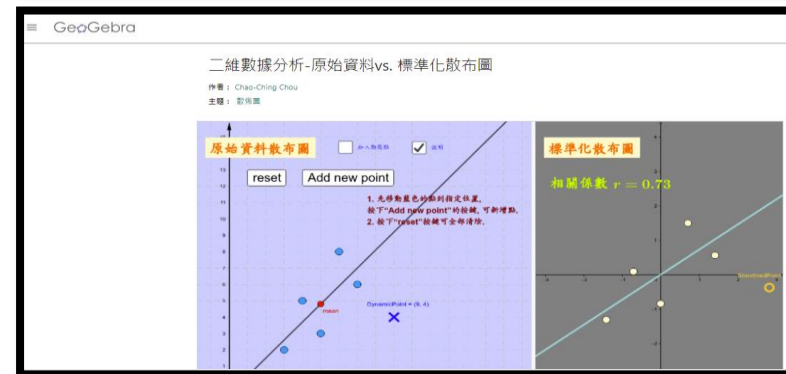
将所有项移到等式左边并化简。  
+ 点击了解更多的步骤...

$$3x^2 - 48 = 0$$

使用二次公式求解。

$$\frac{-b \pm \sqrt{b^2 - 4(ac)}}{2a}$$

代入并替换  $a = 3$ ,  $b = 0$ , 和  $c = -48$  的值到二次方程式中求解  $x$ 。

$$\frac{0 \pm \sqrt{0^2 - 4 \cdot (3 \cdot -48)}}{2 \cdot 3}$$


# Outline

1

- 回顧：近年來國際上結合科技與數學教育的研究議題有何發展趨勢？

2

- 現在：目前台灣的數學教育在推動科技與數學教育結合上，有哪些成果？

3

- 展望：邁向數學教育4.0，我們還需要哪些努力？

# 先前研究中

- ▶ 主要使用的科技類型為何?
- ▶ 如何與適切教學法結合?
- ▶ 是否有效?

# 回顧先前文獻中結合科技與數學 教育的研究議題與趨勢

- Cheung, A.C.K., & Slavin, R.E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review*, 9, 88–113.

**Year of publication: 1960~2011**

**Type of technology : Computer-Managed Learning (CML) (7), Comprehensive Models (8), and Supplemental CAI Technology (37)**

**The overall weighted effect size is +0.16**

- Young, J. L. (2017). Technology-enhanced mathematics instruction: A second-order meta-analysis of 30 years of research. *Educational Research Review*, 22, pp.19-33.

**Year of publication: 1986~2015**

**Type of technology : computation enhancement technologies, instructional delivery enhancement technologies, and presentation and modeling enhancement technologies**

**The mean effect sizes were 0.47, 0.42, and 0.36**

- Bray, A. & Tangney, B. (2017). Technology Usage in Mathematics Education Research - A Systematic Review of Recent Trends. *Computers & Education*, 114, pp.255-273.

**Year of publication: 2012~2016**

Technology	Learning Theory	SAMR Level	Purpose
Collaborative by Design	Behaviourist	Augmentation	Change in Attitude
Dynamic Geometry Environment	Cognitive	Modification	Improve Performance
Multiple Linked Representations	Constructivist	Redefinition	Improve Conceptual Understanding
Outsourcing – Computation	Social		Skills-focused
Outsourcing – Content	Constructivist		Support Teachers
Programming Tools	Constructionist		Collaboration and Discussion
Toolkit			

Fig. 4. Components of the classification.

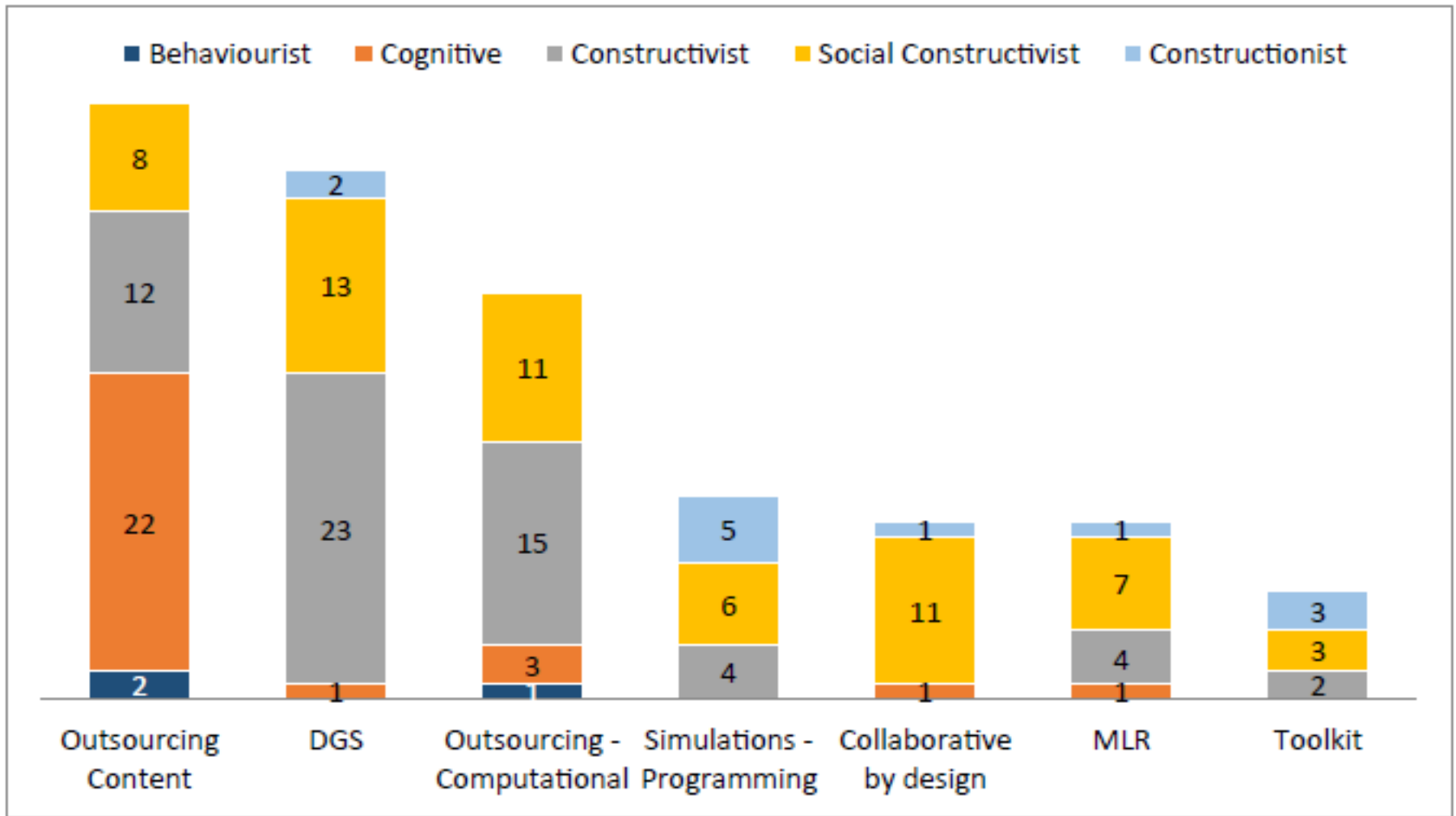


Fig. 7. Technology v Learning Theory.

Bray, A. & Tangney, B. (2017). Technology Usage in Mathematics Education Research - A Systematic Review of Recent Trends. *Computers & Education*, 114, pp.255-273.

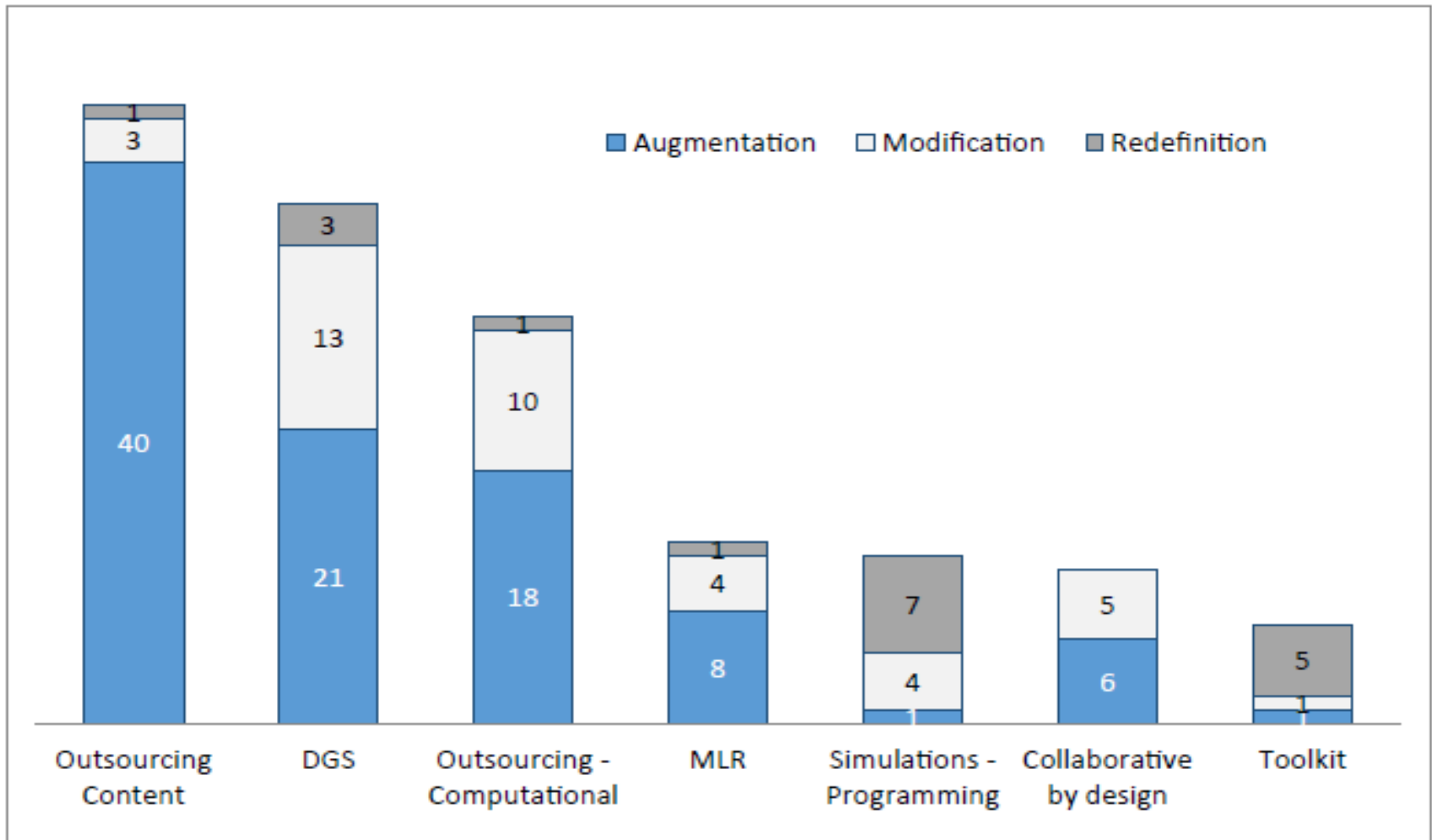


Fig. 8. Technology v SAMR.

Bray, A. & Tangney, B. (2017). Technology Usage in Mathematics Education Research - A Systematic Review of Recent Trends. *Computers & Education*, 114, pp.255-273.



- ▶ Rakes, C. R., Ronau, R. N., Bush, S. B., Driskell, S. O., Niess, M. L., & Pugalee, D. K. (2020). Mathematics achievement and orientation: A systematic review and meta-analysis of education technology. *Educational Research Review*, 100337.
- ▶ **Type of technology :**
- ▶ **Calculators( 計算機 ):** scientific, graphing calculators, calculator programming, computer algebra systems (CAS), dynamic geometry, simulation, and statistics.
- ▶ **Probeware(探測裝置):** data collection devices such as Calculator Based Laboratory (CBLTM), Computer Based Ranger (CBRTM), motion detectors, and other specialized sensors (e.g., temperature, pressure, velocity).
- ▶ **Software(教學軟體):** dynamic geometry, graphing, algebra, statistics, statistics instruction, spreadsheet, presentation, applets, games/puzzles, testing, tutorial, student response systems, and interactive whiteboard.
- ▶ **Hardware(資訊硬體):** laptops, classroom computers, and computer labs.
- ▶ **Internet technologies(網路科技):** included online manipulatives and applets, distance learning, online games/puzzles, online testing, online tutorial, websites, WebQuests, Wiki spaces, social media (e.g., Facebook, Twitter), video conferencing, document or video sharing, and blogs.

Rakes, C. R., Ronau, R. N., Bush, S. B., Driskell, S. O., Niess, M. L., & Pugalee, D. K. (2020). Mathematics achievement and orientation: A systematic review and meta-analysis of education technology. *Educational Research Review*, 100337.

- Six broad categories of technology use were identified in the literature: instruction enhancement, computation, support for active learning, tutorial, assessment, and technology-rich environments.
- Technology is generally considered effective to the degree it is used in a way that improves student conceptual understanding of mathematics and encourages mathematical reasoning and communication.

# 科技融入數學教育是有效的嗎？

## 美國教育部「邁向顛峰計畫」證實以科技實現個人化學習有助於提升學習成效

EDUCATION WEEK TEACHER DIGITAL DIRECTIONS MARKET BRIEF TOPSCHOOLJOBS SHOP ADVERTISE

December 21, 2015

EDUCATION WEEK [LOGIN](#) | [REGISTER](#) | [SUBSCRIBE](#) [Get 2-Week FREE Trial >>](#)  
[Sign Up for FREE E-Newsletters](#)

EDUCATION WEEK TEACHER DIGITAL DIRECTIONS MARKET BRIEF TOPSCHOOLJOBS SHOP ADVERTISE

December 21, 2015

EDUCATION WEEK [LOGIN](#) | [REGISTER](#) | [SUBSCRIBE](#) [Get 2-Week FREE Trial >>](#)  
[Sign Up for FREE E-Newsletters](#)

Taking Stock of **Personalized Learning**  Complete Coverage >

### District's Ambitious Personalized Learning Effort Shows Progress



Kayla-Meeks Cook, a 6th grader, gets help from teacher John Williams at Whittemore Park Middle School in Conway, S.C. The students were using their iPads to pick out recipes and calculate ingredients as part of a skills task. —James Jason Lee for Education Week

By Michelle R. Davis

Published in Print: October 22, 2014, as Adaptive Testing Guiding Teachers' Instruction

Taking Stock of **Personalized Learning**  Complete Coverage >

### Adaptive Testing Shaping Instruction

Some districts are finding that assessments have value that extends well beyond getting a reading of students' test scores







By Robin L. Flanigan

At the Waukesha STEM Academy-Saratoga Campus, students can find themselves taking adaptive tests once a day, or once a month, depending on the subject matter and how quickly they work through the curriculum. But the Wisconsin charter school found the customized assessments that adjust the level of difficulty based on the responses of the test-taker so beneficial that school leaders siphoned funds away from a full-time educational position to buy more of this personalized testing.

Students have performed well on the adaptive tests, which appears to have translated to state testing. Less than 5 percent scored in the 24th percentile or lower on state tests, compared with nearly 20 percent five years ago when the school was not using adaptive testing tools.

But it was an inadvertent benefit of adaptive testing that demonstrated to educators at the 336-student school just how individualized their instructional approach could be,

ARTICLE TOOLS

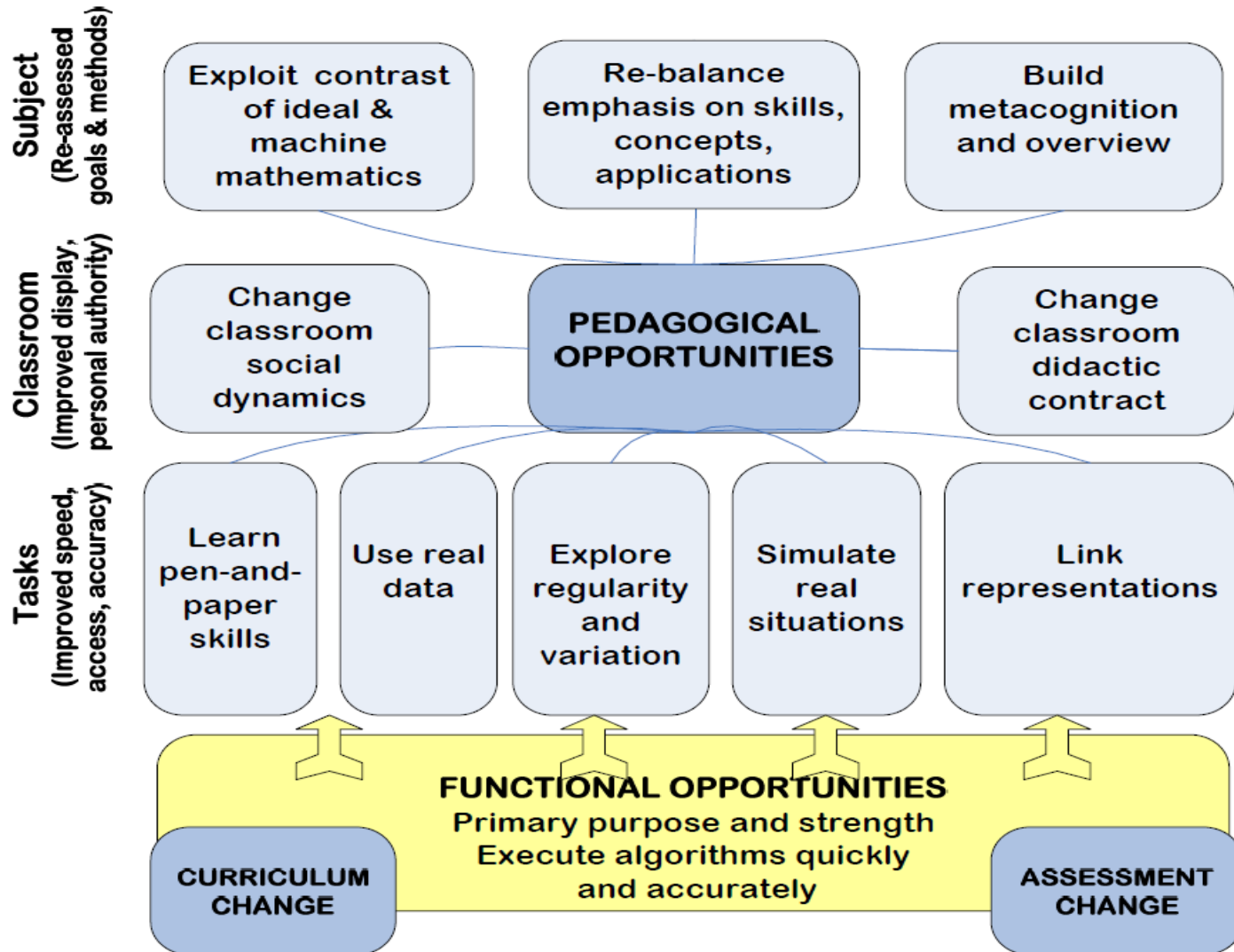
-  [Printer-Friendly](#)
-  [Email Article](#)
-  [Reprints](#)
-  [Comments](#)
-  [Tweet](#)
-  [Like](#)

EDUCATION WEEK  
GROUP ONLINE ACCESS

# 科技融入數學教育是有效的嗎？

- ▶ 美國非營利性研究機構RAND曾通過對全美62所K-12學校、近1萬名學生的研究，結果發現：在實施個性化學習的學校，學生平均閱讀與數學能力顯著提升，且平均提升程度顯著高於沒有實施個性化學習的學生（引自 <https://kknews.cc/education/y5jk4xk.html>）
- ▶ The use of technology has the potential to improve the teaching and learning of mathematics, leading to gains in higher order thinking skills (Wenglinsky, 1998) as well as student achievement and self-efficacy (Mistretta, 2005).
- ▶ Findings from the meta-analyses studies (Cheung and Slavin, 2013; Li and Ma, 2010; Rosen and Salomon, 2007; Tamim et al., 2011; Huscroft-D'Angelo, Higgins, & Crawford, 2019) reported positive, small to moderate overall effect sizes when using technology in mathematics education.

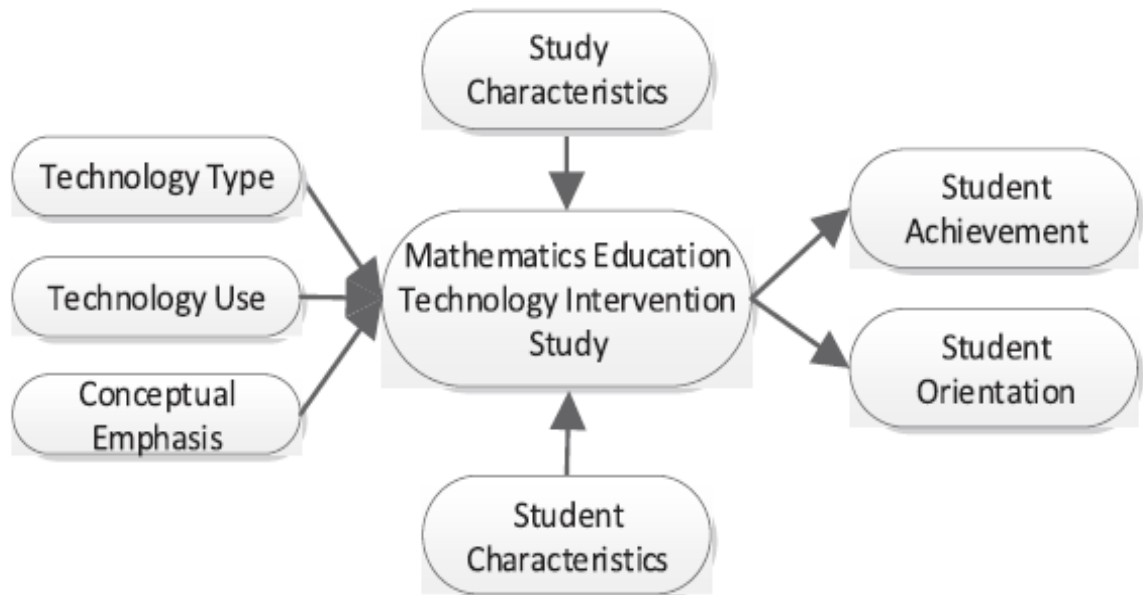
# Pierce & Stacey (2010)



Pedagogical map for mathematics analysis software

# 歸納先前文獻中 結合科技與數學教育的研究趨勢

- ▶ 科技型態多元化，從靜態→動態→互動式、探究式
- ▶ 新興科技與人工智慧的引入，MOOCS與智慧家教系統(ITS)的興起，個別化、適性化科技逐漸成為主流
- ▶ 建議更完整探討各項影響因子



# 呼應2020 Horizon Report之預測

- ▶ Top six emerging technologies with significant impact on the present and future of teaching and learning
  - Adaptive Learning Technologies
  - AI/Machine Learning Education Applications
  - Analytics for Student Success
  - Elevation of Instructional Design, Learning Engineering, and UX Design in Pedagogy
  - Open Educational Resources
  - XR (AR/VR/MR/Haptic) Technologies

<https://library.educause.edu/resources/2020/3/2020-educause-horizon-report-teaching-and-learning-edition>

# 2021 Horizon Report之預測

---

- ▶ Artificial Intelligence (AI)
- ▶ Blended and Hybrid Course Models
- ▶ Learning Analytics
- ▶ Microcredentialing
- ▶ Open Educational Resources (OER)
- ▶ Quality Online Learning

<https://library.educause.edu/resources/2021/4/2021-educause-horizon-report-teaching-and-learning-edition>



# 目前台灣的數學教育在推動新興科技與數學教育結合上，有哪些成果？

人工智慧在數學教育的應用

VR與AR在數學教育的應用

運算思維與數學教育的結合

數學教育的巨量資料分析



# AI for Education

- ▶ Support Teachers(即時掌握學習狀態與方式因材施教)
- ▶ Support Students (AI機器人學伴)
- ▶ Meet a Variety of Student Needs (學習推薦系統)
- ▶ Automate Grading (自動評閱、智慧評測)
- ▶ Allow Teachers to Act as Learning Motivators (傳授知識和技能服務由「機器人教師」代勞；老師能有更多時間和學生互動，深度關懷)
- ▶ Provide Personalized Help (課後自動化輔導)
- ▶ Identify Weaknesses in the Classroom (學習診斷、行為分析、情緒與態度分析)

# 數學適性學習平台的發展

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合  
數學教育  
的巨量資料  
分析

**ALEKS**  
LOGIN ABOUT US NEWS CAREERS CONTACT US SUPPORT FEEDBACK

WHAT IS ALEKS? COURSE PRODUCTS HIGHER EDUCATION K-12 INDEPENDENT USE & HOMESCHOOL

Log In Name  
Password  
LOG IN  
Forgot your login info?  
New Student? SIGN UP NOW  
TAKE A TOUR FREE TRIAL

Never before have I seen a mathematical teaching program that builds such confidence in students. ALEKS creates independent learners."  
—Professor Owen, NE | Lone Star College, TX

ALEKS HIGHER EDUCATION  
Instructors // Administrators // Students  
▶ Success Stories ▶ Implementations

ALEKS K-12  
Teachers // Administrators  
▶ Success Stories ▶ Implementations ▶ Buy ALEKS

ALEKS INDEPENDENT USE  
Teachers // Students // Homeschoolers  
▶ Success Stories ▶ Implementations ▶ Buy ALEKS

THE USE OF  
PRIVACY

ALEKS is a registered trademark of ALEKS Corporation.  
©2017 ALEKS Corporation. All rights reserved.

**Carnegie Learning**  
Resource Center Software Login Search

Learning Solutions Resource & Support Research & Results About Us Careers Contact Us

Increase Students' Math Knowledge  
Personalized, student-centered math instruction for grades 6-12. Proven to nearly double a typical year's worth of learning.  
Read More →

With Carnegie Learning, I saw amazing growth in my students' ability to precisely explain HOW they arrived at an answer and to justify their method. The depth of understanding that they gained was tremendously greater than I have seen in previous years.

Introducing MATHia X  
M<sup>X</sup>  
MATHia X  
The best math in the galaxy.

Donna Owens  
Lanier County Schools, GA

**iKnow!**  
Features Pricing Log In Start a free trial

Learn a language now.  
Remember it forever.

Try it out  
Try 5 sessions for free

Our Learning Science Does the Heavy Lifting for You.

iKnow's Memory Bank gives you a constant snapshot of your memory. Visualize your progress for every item. See exactly the best time to review. Be confident in your mastery.

**eou 因材網·學習拍 Adaptive Learning**

因材網及適性教學計畫網址已於110年1月更新 更多消息

中小學數位教材  
因材網

最新消息  
因材網及適性教學計畫網址已於110年1月更新  
2021-04-23  
自主學習新功能上線囉!  
2021-04-01

因材網居家線上學習資源(分科安排5/17-5/28課程單元)  
2021-05-19

活動資訊與帳號申請 (請師名單)  
適性教學學校甄選  
自主學習專區  
分享與交流  
操作手冊  
常見問題

圖中小科技輔助自主學習推動計畫  
中小學數位學習深耕推動計畫  
21世紀核心素養教師教學能力提升計畫  
高中職科技輔助自主學習推動計畫

**KNEWTON**  
OUR APPROACH ABOUT RESOURCES SIGN IN BUILD WITH KNEWTON

A Path for Every Student  
Knewton is now offering affordable, adaptive course solutions in higher education

LEARN MORE >

**One on One for Everyone**

Students have different skills and needs. Our pioneering approach draws on a student's own history, what other students do, and decades of research into how people learn to improve future learning experiences. With Knewton, meet each student where they are and lead them to their own best path through the material. Knewton can work with many subjects, age groups, languages, and pedagogical approaches.

Discover the Knewton difference >

# 計畫團隊

## 全國推動



國立臺中教育大學  
教育資訊與測驗統計研究所  
郭伯臣教授

## 國中小團隊

### 數學領域

國立臺中教育大學  
教育資訊與測驗統計研究所  
施淑娟教授



### 國語文領域

國立臺中教育大學  
語文教育學系  
楊裕賢 副教授



### 自然科學領域

國立中央大學  
網路學習科技研究所  
黃翰淵 副教授



### 英語文領域

國立臺中教育大學  
英文學系  
范莎惠 助理教授



## 高中團隊

### 數學領域

國立臺灣師範大學  
數學系  
左台益教授



### 國語文領域

國立臺中教育大學  
語文教育學系  
楊裕賢 副教授



### 英語文領域

國立彰化師範大學  
英文系  
張善賢教授



# 因材網-數位學習內容

	九年一貫課程				十二年國教課綱						
領域	數學	國語文	自然科學	理化	數學	國語文	自然科學	生物	理化	地科	英文
範圍	一至九年級	一至九年級	三至六年級	八至九年級	一至三、七至十一年級	一至三、七至十二年級	三年級	七年級	八年級	九年級	三至十一年級
知識節點	1,003	1,115	472	443	1,201	1,079	46	350	23	170	2,231
教學影片	1,003	1,115	472	443	1,201	1,079	46	350	23	170	2,231
診斷測驗試題	6,006	10,537	3,776	1,772	5,851	10,621	184	1,400	92	680	8,984
互動式教學元件	80	10	10	0	0	0	0	0	0	0	0
動態評量教學元件	2,120	40	40	0	2,280	0	0	0	0	0	0

數據：110年10月13日

# 數學素養導向試題的研發與應用

NEW 操作介紹

課程總覽

指派任務



## 國小

數學

數學

數學 自然 英語文

數學 自然 英語文

## 國中

### 七年級

國語文 數學 英語文  
生物

### 八年級

國語文 數學 英語文  
理化

### 九年級

國語文 數學 英語文  
理化 地科

## 特色專區

### 知識結構星空圖

數學 國語文 自然 英語

### 學科素養

數學 國語文 自然

### 學習扶助

科技化評量 縣市學力檢測

### 特色課程

21世紀核心素養

### 互動學習

【箱內取球】1/3

有 $a$ 顆黑球 $b$ 顆白球在同一個箱子裡面，箱子外面有足夠多的黑球跟白球，每次從箱內取出兩球，再從箱外拿1球放入箱內，規則如下:若取出兩球同色(黑黑、白白)補一白球放入箱內，若取出兩球異色(1黑1白)補1黑球放入箱內，直到箱內只剩1球動作就停止。

你可以使用右方報表改變一開始黑球及白球個數，按下模擬鍵電腦會模擬一輪取球過程。請用模擬來幫助你評估以下的敘述:

敘述	正確	錯誤
1.一開始如果是1黑1白的情況，最後1球一定是黑球	<input type="radio"/>	<input type="radio"/>
2.一開始如果是3黑2白的情況，最後1球一定是黑球。	<input type="radio"/>	<input type="radio"/>
3.一開始是3黑3白的情況，最後1球是黑球和白球的機會一樣高。	<input type="radio"/>	<input type="radio"/>
4.一開始是1黑3白的情況，最後1球是白球的機會比較高。	<input type="radio"/>	<input type="radio"/>

以下是一個模擬畫面，顯示箱內黑球白球個數的變化：

次數	箱內黑球	箱內白球	
一開始箱內的球	<input type="text" value="2"/>	<input type="text" value="1"/>	<input type="button" value="模擬"/>
第1次取球後	0	2	
第2次取球後	0	1	

這一題結合模擬器，進行機率問題之探究

# 數學智慧家教系統

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合  
數學教育的  
巨量資料  
分析

The Cognitive Tutor interface is divided into several panes. The top-left pane, titled 'Scenario', contains a word problem about a rock climber and four questions. The top-right pane, titled 'Milton Avery's skills', lists skills like 'Entering a given identifying units' and 'Finding X, any form'. The middle-right pane, titled 'Solver', shows the algebraic steps to solve for T:  $67 + 2.5T = 92$ ,  $-67$  from both sides,  $2.5T = 25$ ,  $\frac{2.5}{2.5}$  from both sides, and  $T = 10$ . The bottom-left pane, titled 'Worksheet for Problem BH1T20', contains a table with columns 'TIME' and 'HEIGHT' and rows for 'Expression', 'Question 1', 'Question 2', 'Question 3', and 'Question 4'. The bottom-right pane shows a graph with a grid and a point plotted at (10, 92). A 'Hint' dialog box is open, stating: 'You know that the climbing time is 75 seconds. Convert 75 seconds to minutes.'

Cognitive Tutor

The ActiveMath interface shows an 'Exercise' window for the problem 'Compute  $\frac{2}{7} + \frac{1}{2}$ '. The solution is shown in a series of steps: 1. 'Expand'  $\frac{2}{7}$  expanded by 2 equals  $\frac{4}{14}$ . 2. 'Expand'  $\frac{1}{2}$  expanded by 7 equals  $\frac{7}{14}$ . 3. 'Add'  $\frac{4}{14} + \frac{7}{14} = \frac{11}{14}$ . The final 'Result' is  $\frac{11}{14}$ . At the bottom, a green bar contains a confidence slider set to 73%.

ActiveMath



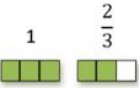
# 數學智慧家教系統

目前位置：智慧家教學習系統


前位置：智慧家教學習系統 > 回單元列表

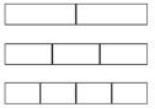
老師想用彩帶佈置教室，一條彩帶長  $1\frac{2}{3}$  公尺，  
如果老師買了 5 條，請問共有多少公尺？


提示

$1\frac{2}{3}$  這樣表示 

老師回應：算算看！先列下算式看看！  
\*學生回答：000 20  
老師回應：請拖出二異區的積木來~  
\*學生回答：1.5  
老師回應：1  $\frac{2}{3}$   $\times$  5 = ?

操作區 


工具區 

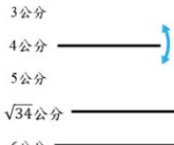
 送出




老師：同學，請點選黑框拖拉至操作區作圖  
益智：我也來試試！

已知直角三角形的兩邊長是 3 和 5，請拖拉工具區的線段至操作區，畫出可能的直角三角形。

操作區 

工具區 

 送出

$\lim f(x) = L$

式，將上式定義的極限值函，輸入在下方的作答區內。  
化的方式，將極限定義之含義完整輸入，除了上式所使用之符  
\、自變數 x、實數值 a、實數值 L 以外，請勿使用其他數字

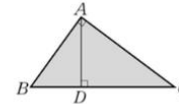
老師回應：請用文字的方式，依據題目  
目的問題，將答案輸入在做答區

## Math ITS

形 ABC 中， $\angle BMC$  是直角，已知  $AB=9$ ， $AC=12$

求  $\overline{BC}$  的長度

兩股分別為  $\overline{AC}$ 、 $\overline{BC}$   
可利用畢氏定理求解



老師：同學，請利用作答區工具作答，請不要用鍵盤作答喔！

益智：同學，我們一起來作答！

操作區 

工具區 

 送出



# 數學解題智慧工具的發展

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

數學教育  
的巨量資料  
分析

Trigonometry

Mathway  
a Chegg® service

How can I help you?

Enter a problem...

$f(x)$				$y$				$x^2$			
(	)		[	]	$\sqrt{\quad}$	$\sqrt[3]{\quad}$	$\geq$	sin	$^\circ$	$f(x)$	$i$
$x$	7	8	9	$\frac{\square}{\square}$	$\square^2$	$\square^3$	$\leq$	cos	$\theta$	ln	$e$
$y$	4	5	6	/	$\wedge$	$\times$	$>$	tan	$\pi$	log	$\log_{\square}$
$z$	1	2	3	-	+	$\div$	$<$	$\cot$ $\sec$ $\dots$	$\triangle$	$\square$ $\square$	$\square$ $\square$
abc	,	0	.	%	$\_$	=	$<$	$>$	$\times$	$\leftarrow$	

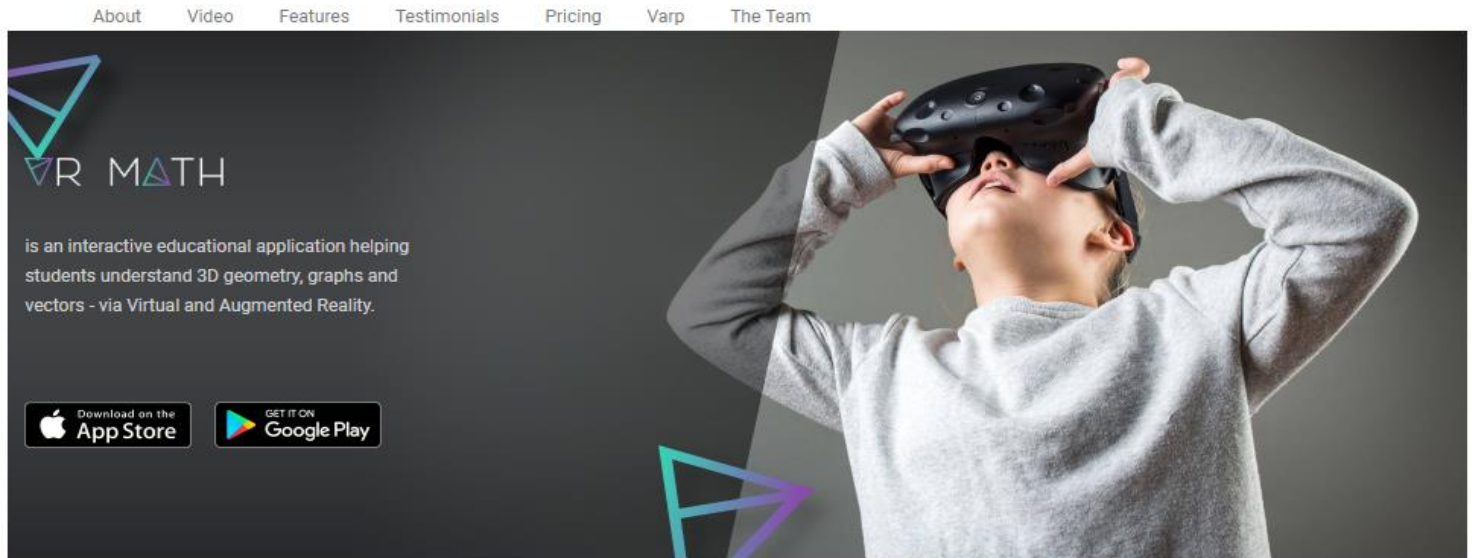
# VR in MATH

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

數學教育  
的巨量資料  
分析



<https://www.youtube.com/watch?v=E-ZlqaTMIsU>

許一珍、林仁智、林奕辰、張文馨、陳致錦（2018年11月）。探討虛擬實境應用至高中數學科之學習成效—以三元一次聯立方程式單元為例。載於國立中央大學（主編），TANET2018 臺灣網際網路研討會（2507-2511頁）。台灣：國立中央大學。

doi:10.6861/TANET.201810.0464

National Taichung University of Education  
Graduate Institute of Educational Information and Measurement

# AR in MATH

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

數學教育  
的巨量資料  
分析

Research and Practice in Technology Enhanced Learning  
Vol. 7, No. 3 (2012) 153–173  
© Asia-Pacific Society for Computers in Education

## COLLABORATIVE MATHEMATICAL INQUIRY WITH AUGMENTED REALITY

HÅKAN SOLLERVALL

*Halmstad University and Linnaeus University,  
Halmstad and Växjö, Sweden  
hakan.sollervall@hh.se*

In this paper we describe and reflect on the design of a mathematical learning activity developed in collaboration between teachers, researchers and technical developers. By making use of augmented reality (AR) as a technology supporting augmentation of a real-world projection with computer-generated images, we have designed an activity that promotes unique action and learning trajectories. These trajectories require the learners to engage in interactive-constructive actions that involve and stimulate the development of their self-regulatory skills by inviting them to vary and coordinate across the contextual affordances of the technologies and the physical resources in the classroom. Our learning activity is designed as a collaborative guided inquiry, implemented in a regular classroom and involved mathematical problem solving in relation to the geometric concept

王俊堯、林英志（2019）。應用擴增實境輔助學習系統於微積分教育之成效評估。高等教育研究紀要，11。25-38。

AR/VR/MR

- ▶ AR (Augmented Reality): 真中有假
- ▶ VR (Virtual Reality): 全都是假
- ▶ MR (Mixed Reality): 似真似假

# Math + Coding project

人工智慧  
在數學教育  
的應用


VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

數學教育  
的巨量資料  
分析

- ▶ 加拿大的社會科學與人文研究委員會 (Social Sciences and Humanities Research Council, SSHRC) 與數學科學研究費爾茲機構 (The Fields Institute for Research in Mathematical Sciences, FIRMS) 所資助的「數學教育中的運算思維」

Vol 3, Issue 1   Vol 2, Issue 3   Vol 2, Issue 2   Vol 2, Issue 1   Vol 1, Issue 3   Vol 1, Issue 2   Vol 1, Issue 1



## Math + Code 'Zine


Exploring math through code

Home   Issues   About

Volume 3 Issue 1 (March 2018)

[Tweet](#)

Editor's Message



Computational Thinking Activities: Enacting Leveled Concepts — *Immaculate K. Namukasa*

Here is one categorization that is applicable to designing computational thinking learning opportunities for students in the elementary grades: Introductory, Intermediate and Advanced. At each of these levels, students may be seen to engage in computational thinking activities in which domain-specific concepts, such as mathematics concepts, are enacted, practiced or consolidated.

Read more


Follow us on Twitter!

[Follow @mathncode](#)

Subscribe to Math + Code

Name: \_\_\_\_\_  
Email: \_\_\_\_\_  
[Send](#)

Featured Articles




Review Article for Edison Edblocks — *Melanie Drummond*

At St. John School in Guelph, Ontario there are quite a few students who had the opportunity to learn and use the coding program "Scratch". A Coding Club was formed! These coding coaches are presently in Grade 4. They were interested in trying something a little different and had the opportunity to try a brand-new program involving Edison robots called Edblocks.

## Growing Patterns - Game

VIEW:  Code  Flowchart  Text



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

N = 1

repeat ?

show 3 x N

N = N + 1

TEST ✓   NEXT →   SHOW 🗨️

Question: 1/5  
Current Score: 0/15  
High Score: 0/15  
Level: 1/3   Goal: 3/15

Before the start of each game, set your level and goal (bottom-left). If you meet this goal, you'll get a math surprise! Change values in the code to get the highlighted numbers. Click TEST to test your answer! First Try - 3 Points; Second try - 2 Points; Third try - 1 Point -- Have fun!

# ScratchMaths

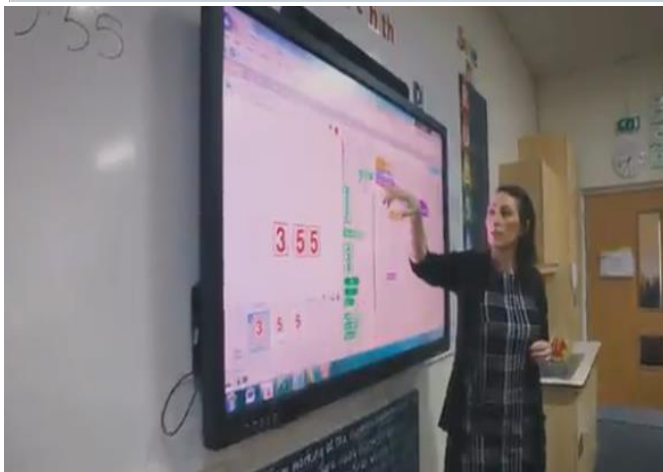
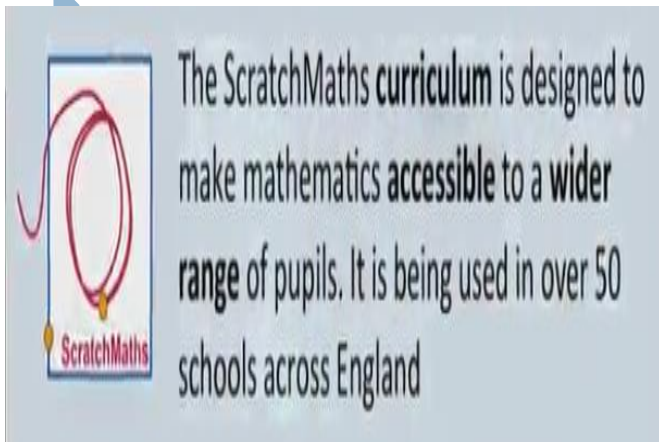
building mathematical knowledge with programming

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

數學教育  
的巨量資料  
分析



```
go to x: 0 y: 0
clear
pen down
repeat 20
  draw circle
  turn 20 degrees
  change pen color by 25
```

```
define draw circle
repeat 90
  move 4 steps
  turn 4 degrees
```

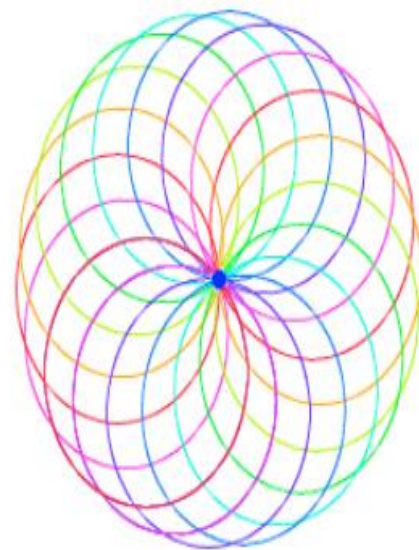


Figure 2. Creating a circles pattern in Scratch.

# 運算思維 X 數學教育

人工智慧  
在數學教育  
的應用

VR與AR  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

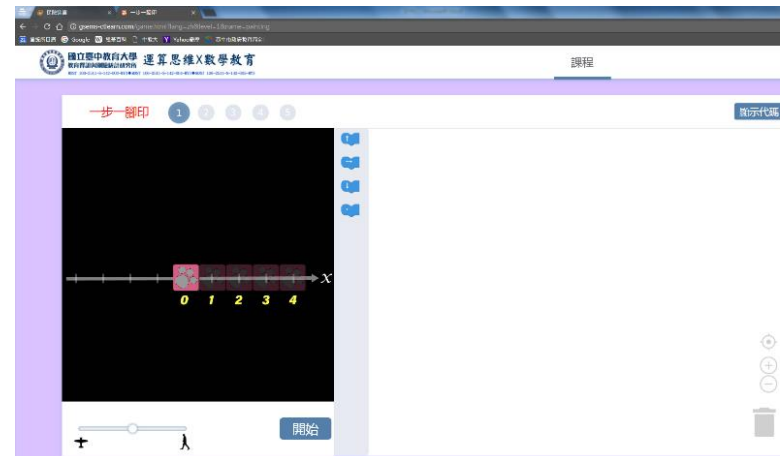
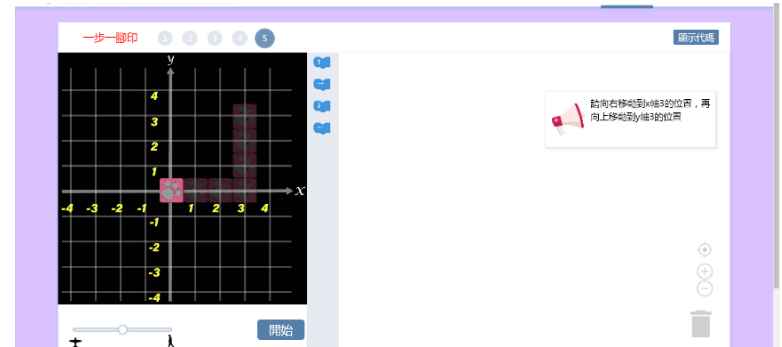
數學教育  
的巨量資料  
分析

- ▶ 李政軒(2018)：結合運算思維之平面直角坐標系單元課程



# 一步一腳印

- ▶ 利用運算思維常見的程式區塊「前進」搭配坐標概念。
- ▶ 引入坐標程式區塊
  - 當 $x$ 軸上的方塊向左移動為正數、向右移動為負數；
  - 在 $y$ 軸時亦同，向上移動為正數、向下移動為負數。





# 探險迷宮

- ▶ 利用運算思維常見的程式區塊「前進」搭配坐標概念。
- ▶ 需結合運算思維迴圈概念。
- ▶ 結合運算思維迴圈概念與坐標程式區塊
  - 「 $x+1$ 」、「 $x-1$ 」、「 $y+1$ 」、「 $y-1$ 」。

國立華中教育大學 運算思維X數學教育  
課程

探險迷宮 1 2 3 4 5



Y=1  
Y=2  
Y=3  
Y=4


開始

顯示代碼

貓咪的坐標位置(3,3)，請使用游標木移動貓咪至寶藏的位置！

國立華中教育大學 運算思維X數學教育  
課程

探險迷宮 1 2 3 4 5



Y=1  
Y=2  
Y=3  
Y=4

開始

顯示代碼

貓咪的坐標位置(-3,-3)，請使用游標木，沿著道路走到坐標位置(3,3)，才能順利找到寶藏哦！

# Learning analytics and math learning

人工智慧  
在數學教育  
的應用

運算思維  
與數學教育  
的結合

VR與AR  
在數學教育  
的應用

數學教育  
的巨量資料  
分析

- ▶ Ebner, M. & Pronegg, M. (2015). Use of Learning Analytics Applications in Mathematics with Elementary Learners. *International Journal of Academic Research in Education*, 1(2), 26-39. DOI: 10.17985/ijare.37247
- ▶ Taylor Martin, Carmen Petrick Smith, Nicole Forsgren, Ani Aghababyan, Philip Janisiewicz & Stephanie Baker(2015) Learning Fractions by Splitting: Using Learning Analytics to Illuminate the Development of Mathematical Understanding, *Journal of the Learning Sciences*, 24:4, 593-637, DOI: [10.1080/10508406.2015.1078244](https://doi.org/10.1080/10508406.2015.1078244)
- ▶ Dani, A. (2016). Students' patterns of interaction with a mathematics intelligent tutor: learning analytics application. *International Journal on Integrating Technology in Education (IJITE)*, Vol.5, No.2.
- ▶ Xia, M., Wei, H., Xu, M., Lo, L. Y. H., Wang, Y., Zhang, R., & Qu, H. (2019). Visual analytics of student learning behaviors on K-12 mathematics e-learning platforms. *arXiv preprint arXiv:1909.04749*.
- ▶ Zhuhadar, L., Daday, J., Marklin, S., Kessler, B., & Helbig, T. (2019). Using survival analysis to discovering pathways to success in mathematics. *Computers in Human Behavior*, 92, 487-495.
- ▶ 葉秋呈 (2006)。隸屬度函數應用於 e 世代科技大學學生數學之學習分析。 *管理科學與統計決策*, 3(4), 41-57。

# 未來展望

# The future of education: 教育4.0

- ▶ Transforming the future of education through advanced technology.
- ▶ **教育4.0**是**智慧校園**、**客製化**、**創新化**教與學的時代

## What is Education 4.0?



**Anywhere Anytime**



**Personal**



**Flexible Delivery**



**Peers and Mentors**



**Why/Where not What/How**



**Practical Application**



**Modular and Projects**

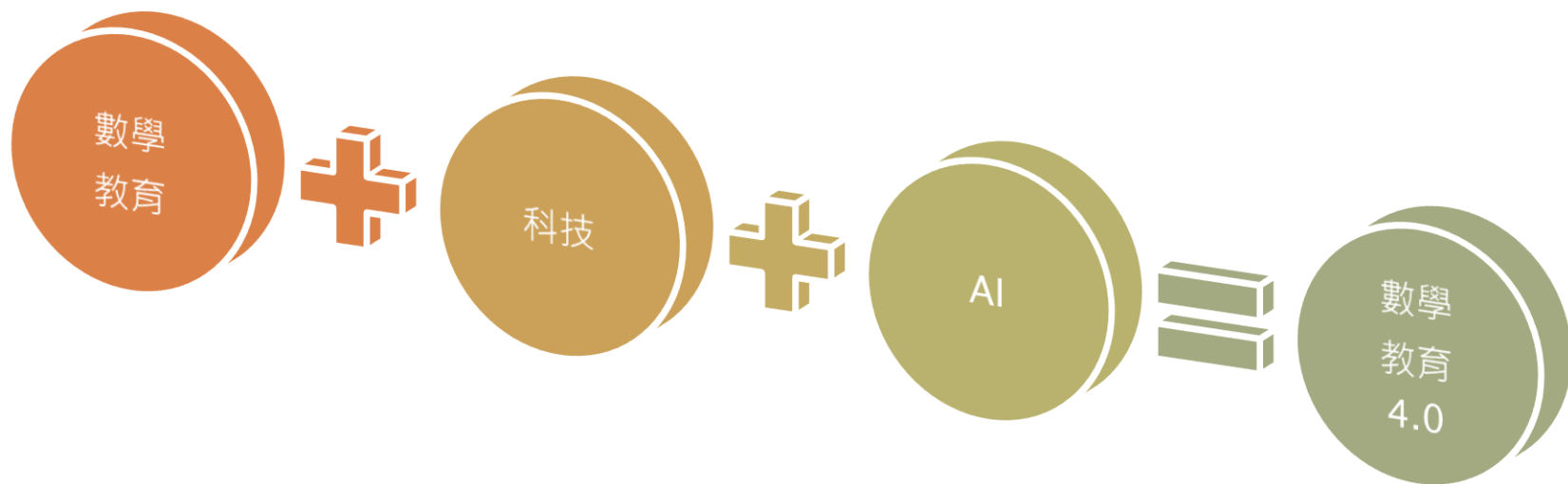


**Student Ownership**

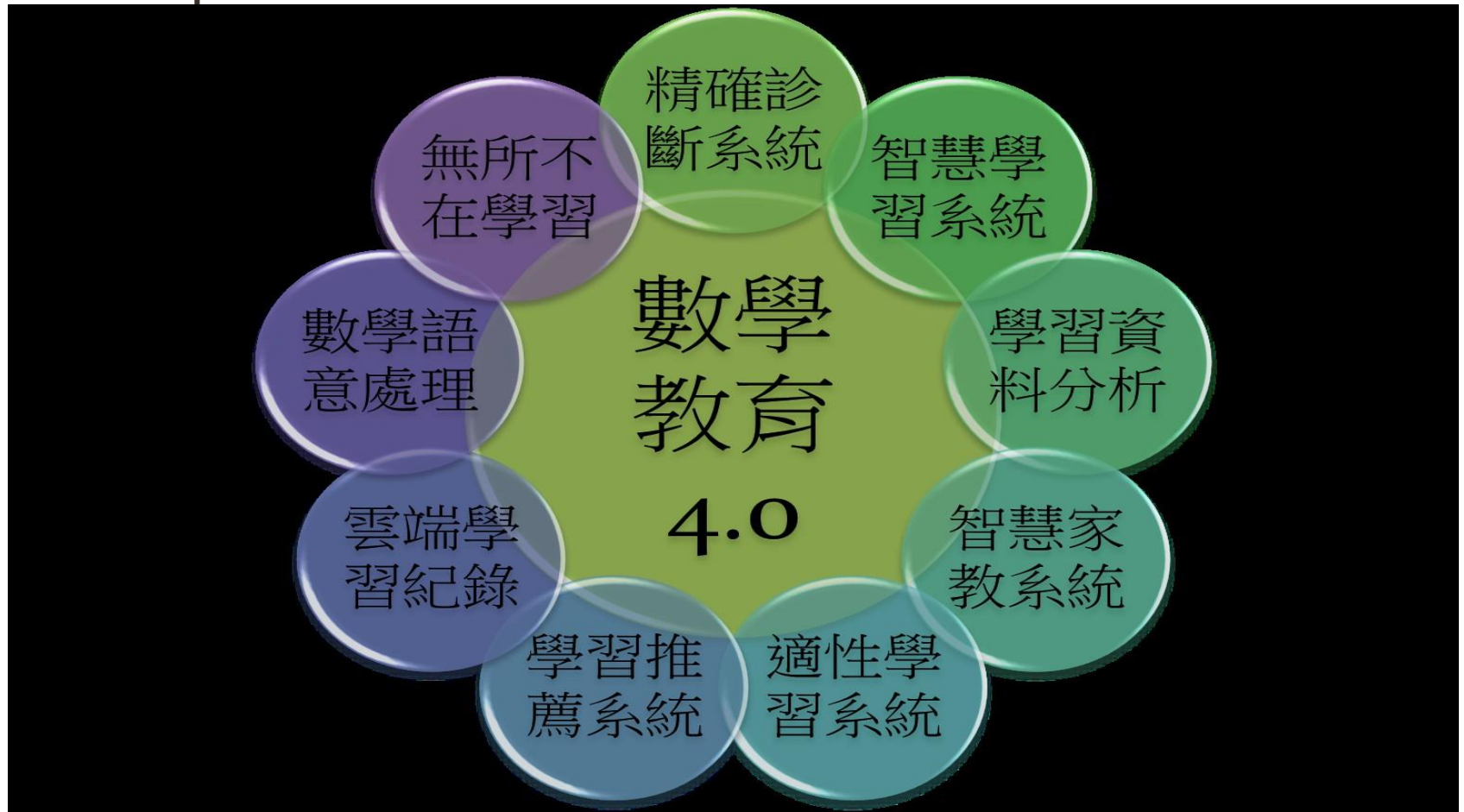


**Evaluated not Examined**

# 數學教育4.0

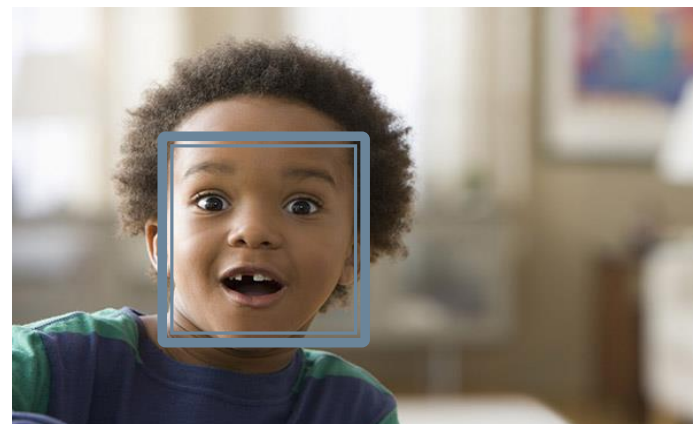
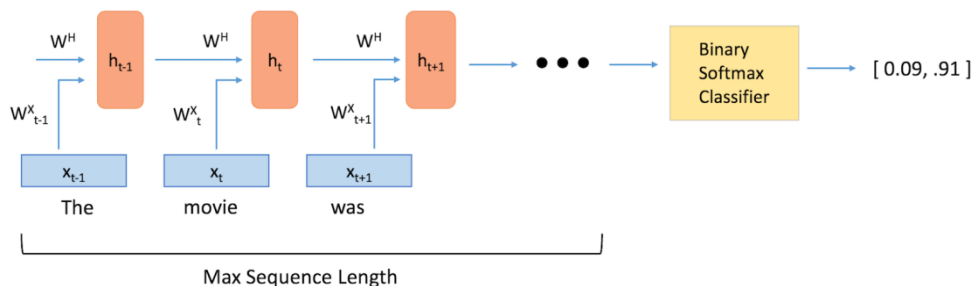


# 邁向數學教育4.0



# 人工智慧及深度學習 在數學教育上的進階發展與應用

1. 利用語意分析進行更精準的數學題意理解分析。
2. 利用深度學習的序列分析進行學生學習軌跡的分析。
3. 利用人工智慧進行教學或解題時學生的情緒偵測與辨識。



# 智慧家教系統與學習分析技術的結合

**Kurt VanLehn**

Last update: April 7, 2017

## Current funded projects

- **FACT: Formative Assessment with Computing Technology:** The intelligence of a tutoring system is used to assist middle school and high school math teachers as they walk around their classroom helping pairs of students explore complex mathematical activities on tablets.
- **Dragoon** To really understand a complex system, nothing beats modeling it, which is why modeling has been prominent in recent standards for math and science. Dragoon allows instructors to author effective instruction in modeling which includes students constructing models with help from an intelligent tutoring system.
- **TopoMath:** Algebra word problems are an early and notoriously difficult introduction to mathematical modelling. The goal of the TopoMath project is to get students to master this difficult skill in 20 hours of instruction. The instruction combines an intelligent tutoring system, a direct-graph notation for equations, an adaptive learning management system, and a variety of in-class activities.



# 可思考的問題

- ▶ 邁向數學教育4.0，我們在理論面、實務面以及政策面還需要哪些努力？
- ▶ 資訊科技發展改變了數學教學的方式，如何檢測現職數學老師準備好了嗎？
- ▶ 教師專業發展及師資培育課程應如何因應，以提升現職與職前數學教師有效運用科技來增進數學學習成效的意願與能力？



感謝聆聽  
敬請不吝指正

